



University of Hradec Králové  
Faculty of Science

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PROCEEDINGS OF  
*3<sup>rd</sup>* INTERNATIONAL PHYSICS DAYS  
of HRADEC KRÁLOVÉ 2024

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# Hradec Králové International Physics Days

3<sup>rd</sup>



## University of Hradec Králové 13<sup>th</sup>-14<sup>th</sup> DECEMBER 2024

**Honorary Chair  
Jan Kříž**

Rector of University of Hradec Králové

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- The inaugural day of the online conference will comprise solely live talks and will solely concentrate on the subject of **BLACK HOLES**.
- The final day of the online conference will feature live or pre-recorded talks on demand **in any field of theoretical physics**.
- All presentations will be delivered via **Zoom**. The relevant link will be distributed on **12/12/2024**.
- All speakers are required to present their talk's title and abstract in advance of the deadline, which is **10/12/2024**.
- All pre-recorded talks must be sent before **10/12/2024**.
- The speaker of the pre-recorded talks must be online during their session. If they are not, their talk will not be played.
- For more information and registration, visit our <http://uni.uhk.cz/ipd/> or contact us by: [conference.ipd@uhk.cz](mailto:conference.ipd@uhk.cz)

	13.12.2024	Local time (CET)	Title
	Jan Křiz	07:45-07:55	Opening Talk
1	Surojit Dalui	08:00-08:30	Chaos and Instability in Black Hole Thermodynamics: Unravelling the Temperature Mystery
2	Reggie C. Pantig	08:30-09:00	Shadow and deflection angle in a Schwarzschild-like metric
3	Arpan K Mitra	09:00-09:30	Spinning Black Hole in Fluid
4	Sanjar Shaymatov	09:30-10:00	On black hole formation in higher dimensions
	<b>BREAK</b>		
5	Javlon Rahimbaev	10:30-11:00	Probing Gravity with Pulsar Deathlines
6	Izzet Sakalli	11:00-11:30	Lorentz-Violating Effects of the Kalb-Ramond Field on Charged Black Hole Thermodynamics and Photon Dynamics
7	Antonina Zinhailo	11:30-12:00	Quasinormal spectrum in the asymptotically safe gravity
8	Jutta Kunz	12:00-12:30	Properties and Ringdown of Rotating Black Holes in EsGB Gravity
9	Roman Konoplya	12:30-13:00	The sound of the event horizon
	<b>BREAK</b>		
10	Ali Övgün	13:30-14:00	Exact Regular Black Hole Solutions with de Sitter Cores and its Shadow cast
11	Piero Nicolini	14:00-14:30	What if black holes are singularity-free?
12	Robert Mann	14:30-15:00	Quantum Superpositions of Black Holes
13	Olexander Zhidenko	15:00-15:30	Ringdown of black holes with Infinite tower of higher-curvature corrections
14	Ahmad Al-Badawi	15:30-16:00	Schwarzschild Black Hole in Galaxies Surrounded by a Dehnen- type DM halo
15	Oleksandr Stashko	16:00-16:30	Observational imprints of solutions from smeared sources

	14.12.2024	Local time (CET)	Title
16	Farokhnaz Hosseinifar	09:00-09:15	Some New Properties of Black Holes in the Quantum Oppenheimer-Snyder Model
17	Fariba Kafikang	09:15-09:30	Investigating The White Dwarf Radius In The Kappa Deformation Formalism Using The Lane-Emden Equation
18	Asma Tahar Taiba	09:30-09:45	Testing Quantum Gravity with Dilute Dipolar Bose gases
19	Abdelhakim Bensaid	09:45-10:00	Challenges in Cosmology: Observing and Modelling the Accelerating Universe
20	Nadjet Benchiheb	10:00-10:15	Analytical solution of the Klein-Gordon equation for nonpolynomial isotonic oscillator potential type using a proper quantization rule
21	Esma Zouaoui	10:15-10:30	Synchrotron Radiation and Self-Absorption Effects in GRB Afterglows
22	Hadjer Didouh	10:30-10:45	Biofilm Formation on API 5L X52 Steel Surfaces Used in Algerian Crude Oil Pipelines and Its Mitigation Strategies
23	Ourida Ourahmoun	10:45-11:00	GPVDM Simulation Of Layer Thickness Effect And Work Temperature On Power Conversion Efficiency Of CH <sub>3</sub> NH <sub>2</sub> PbI <sub>3</sub> Based Perovskite
24	Zohra Mehri	11:00-11:15	Anderson Localization Of BQPs Of Quantum Liquids
25	Abdellah Touati	11:15-11:30	Black Hole Phase Transition And Similarity Between Non-Commutative Schwarzschild Black Hole And AdS-Reissner-Nordstrom Black Hole
26	Hamza Taibi	11:30-11:45	A Scale Invariant Georgi Machacek Model
27	Rabah Oubagha	11:45-12:00	Van der Waals Black Hole in Deformed Space With Maximal Length
28	Mustapha Anis Younes	12:00-12:15	A lightweight and efficient semi-quantum secret sharing protocol.
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29	Zeyneb Taibi	13:15-13:30	The Solitonic Solutions Of Hirota Equation
30	Houri Chaachoua Sameut	13:30-13:45	Breathers Solutions of Nonlinear Hirota Equation with Higher-Order Cubic Nonlinearities, Higher-Order Dispersion, Damping, and External Potential
31	M'hamed Hadj Moussa	13:45-14:00	The Duffin-Kemmer-Petiau Oscillator And The Generalized Momentum Operator
32	Moussa Abbad	14:00-14:15	Movement Of Boson Particles With Different Types Of Potentials In The Default Topological Spacetime
33	Fatima Koudjil	14:15-14:30	Illustrations Of The Post-Gaussian Approximation On Single Species
34	Hadj Mouloudj	14:30-14:45	Evaluation Of The Electric Quadrupole Moment Microscopic in the 58 <N< 82 Region, Xenon
35	Ferhat Tas	14:45-15:00	Dual Vectors For Hamiltonian Systems: Towards A Unified Representation
36	Lakhdar Sek	15:00-15:15	2D Bosonic Oscillator In A Magnetic Field in Noncommutative Anti-de-Sitter Spaces
37	Nergis Heidari	15:15-15:30	A New Approach To Explore Quasinormal Modes Of Black Holes
38	Naouel Chelil	15:30-15:45	Theoretical Investigation of the Bulk Photovoltaic Effect in Mixed Halide Perovskites for Solar Cells
39	Faiza Bettahar	15:45-16:00	Nanofibres For Wound Dressing Application
40	Ali Ozhan Akyuz	16:00-16:15	Modeling the Diffusion of Opinion on Climate Change: A Simplified Practice of Social Physics

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# CHAOS AND INSTABILITY IN BLACK HOLE THERMODYNAMICS: UNRAVELLING THE TEMPERATURE MYSTERY

Surojit **DALUI**

Department of Physics, Postdoc, Shanghai University,  
99 Shangda Road, Baoshan District, Shanghai 200444, China  
0000-0003-1003-8451  
surojtdalui003@gmail.com

## Abstract

Historically, researchers have been interested in the thermal and geometrical characteristics of black hole horizons and their relationship with particle motion. They have discovered that systems exhibit chaotic dynamics in the near-horizon regions. However, the reason for these features and the phenomenological quality of all horizons remain unclear. On the other hand, black hole thermodynamics, based on the analogy between black holes and typical thermodynamical systems, has not been fully understood. This thesis aims to provide a unified explanation that these two characteristics have some intrinsic relationship in the near-horizon region, which can explain why black holes have temperatures.

## References

- [1] S. Dalui, B. R. Majhi and P. Mishra, “Presence of horizon makes particle motion chaotic,” *Phys. Lett. B* **788** (2019), 486-493 [arXiv:1803.06527 [gr-qc]].
- [2] S. Dalui, B. R. Majhi and P. Mishra, “Horizon induces instability locally and creates quantum thermality,” *Phys. Rev. D* **102** (2020) no.4, 044006 [arXiv:1910.07989 [gr-qc]].
- [3] S. Dalui and B. R. Majhi, “Near horizon local instability and quantum thermality,” *Phys. Rev. D* **102** (2020) no.12, 124047 [arXiv:2007.14312 [gr-qc]].
- [4] S. Dalui, B. R. Majhi and T. Padmanabhan, “Thermal nature of a generic null surface,” *Phys. Rev. D* **104** (2021) no.12, 124080 [arXiv:2110.12665 [gr-qc]].

## SHADOW AND DEFLECTION ANGLE IN A SCHWARZSCHILD-LIKE METRIC

Reggie C. **PANTIG**

Physics Department, Mapúa University,  
658 Muralla St., Intramuros, Manila 1002, Philippines  
0000-0002-3101-8591  
rcpantig@mapua.edu.ph

### Abstract

The talk presents a generalized analytic expression for the weak field deflection angle (WDA) in non-asymptotically flat Schwarzschild-like spacetimes, utilizing a generalized Gauss-Bonnet theorem. The formula accommodates deviations from the Schwarzschild metric via constant parameters, enabling the analysis of diverse scenarios including Bumblebee gravity, Kalb-Ramond fields, and black holes immersed in soliton dark matter. The first two cases exemplify distinct Lorentz symmetry breaking mechanisms, yielding novel results and corroborating existing findings. The derived WDA formula facilitates streamlined calculations by allowing direct application of approximations, bypassing intermediate steps. Furthermore, the analysis reveals that the black hole shadow size is solely determined by the metric coefficient associated with the time coordinate. A generalized expression for this parameter is derived, constrained by Event Horizon Telescope (EHT) observations. The work anticipates further generalizations encompassing Reissner-Nordström-like, de Sitter/Anti-de Sitter-like solutions, and higher-dimensional black hole models.

## SPINNING BLACK HOLE IN FLUID

Arpan Krishna **MITRA**

Department of Astrophysics, RA, Aryabhata Research Institute of Observational Sciences ,  
Manora peak, Beluwakhan, Uttarakhand 263002, India  
0000-0002-9865-252X  
arpankmitra@aries.res.in

### Abstract

In this paper, we propose a new Analogue Gravity example - a spinning (or Kerr) Black Hole in an extended fluid model. The fluid model receives Berry curvature contributions and applies to electron dynamics in Condensed Matter lattice systems in the hydrodynamic limit. We construct the acoustic metric for sonic fluctuations that obey a structurally relativistic wave equation in an effective curved background. In a novel approach of dimensional analysis, we have derived explicit expressions for effective mass and angular momentum per unit mass in the acoustic metric (in terms of fluid parameters), to identify with corresponding parameters of the Kerr metric. The spin is a manifestation of the Berry curvature-induced effective noncommutative structure in the fluid. Finally we put the Kerr Black Hole analogy in a robust setting by revealing explicitly the presence of horizon and ergo-region for a specific background fluid velocity profile. We also show that near horizon behavior of the phase-space trajectory of a probe particle agrees with Kerr Black Hole analogy. In fluid dynamics perspective, presence of a horizon signifies the wave blocking phenomenon.

## References

- [1] Spinning black hole in a fluid; 'S. Dalui, A. K. Mitra, D. Mitra, and S. Ghosh, Phys. Rev. D **109**, 064055 (2024).

## ON BLACK HOLE FORMATION IN HIGHER DIMENSIONS

Sanjar **SHAYMATOV**

Institute for Theoretical Physics and Cosmology, Zhejiang University of Technology  
Hangzhou 310023, China

Institute of Fundamental and Applied Research, National Research University TIAME  
Kori Niyoziy 39, Tashkent 100000, Uzbekistan  
0000-0002-5229-7657  
sanjar@astrin.uz

### Abstract

The two main processes of black hole formation are collapse of a matter cloud under its own gravity and accretion of matter onto an already existing gravitating centre. The necessary condition for both the processes to operate is that overall force on collapsing fluid element or on test accreting particles is attractive. It turns out that this is not the case in general in higher dimensions greater than the usual four for collapsing or accreting matter having nonzero angular momentum. Thus, both these processes cannot operate in higher dimensions to form a rotating black hole. Its effect is also reflected in the fact that black hole cannot be overspun even under linear accretion process in  $D > 5$ . The only theory in which this is not the case in higher dimensions is the pure Lovelock gravity, where both these processes could in principle work for formation of black holes.

## PROBING GRAVITY WITH PULSAR DEATHLINES

Javlon **RAYIMBAEV**

University of Tashkent for Applied Sciences, Uzbekistan

Str. Gavhar 1, Tashkent 100149, Uzbekistan

0000-0001-9293-1838

javlon@astrin.uz

### Abstract

Neutron stars are highly magnetized and slowly rotating astrophysical objects with strong gravity near their surfaces. Since there are several effects on the radiation processes around the star, it is impossible to consider all impacts simultaneously. One way to study the methods one by one is by considering them individually under a toy model. The studies on electromagnetic fields and the plasma magnetosphere surrounding neutron stars within gravity theories may help constrain the boundary (upper and lower) values for the gravity parameters using observational parameters of (isolated) pulsars such as period-period derivative and the deathline conditions for radio pulsars through inverse Compton scattering and curvature radiation, which mostly correspond to the X and  $\gamma$  rays band of electromagnetic waves.

# LORENTZ-VIOLATING EFFECTS OF THE KALB-RAMOND FIELD ON CHARGED BLACK HOLE THERMODYNAMICS AND PHOTON DYNAMICS

Izzet SAKALLI

Physics Department, Eastern Mediterranean University  
Famagusta 99628, North Cyprus via Mersin 10, Turkey  
0000-0001-7827-9476  
izzet.sakalli@emu.edu.tr

## Abstract

This seminar explores the impact of Lorentz-violating effects induced by the Kalb-Ramond field on the thermodynamics and photon dynamics of charged black holes. The charged Reissner-Nordström-like black hole solutions in the presence of a background KR field are analyzed, focusing on their geometric and physical properties. Using the Gauss-Bonnet theorem, the deflection angle of photon trajectories is derived in the weak-field approximation. A comparison of geodesics for neutral particles reveals the influence of the Lorentz-violating parameter  $b$  and charge  $Q$  on their trajectories in various planes, such as the equatorial and polar planes. Thermodynamic aspects, including temperature, entropy, and phase transitions, are discussed, highlighting deviations introduced by the KR field. Observational implications in astrophysics, particularly in gravitational lensing and thermal properties of compact objects, are also addressed.

## References

- [1] A. Al-Badawi, S. Shaymatov, I. Sakalli, Eur. Phys. J. C **84**, 825 (2024).
- [2] E. Sucu, I. Sakalli, Exploring Lorentz-Violating Effects of Kalb-Ramond Field on Charged Black Hole Thermodynamics and Photon Dynamics.



## QUASINORMAL SPECTRUM IN THE ASYMPTOTICALLY SAFE GRAVITY

Antonina ZINHAILO

Research Centre for Theoretical Physics and Astrophysics, Institute of Physics, Silesian University in Opava,  
Bezručovo náměstí 13, CZ-74601 Opava, Czech Republic  
0009-0005-8022-4462  
zingaylo@gmail.com

### Abstract

Asymptotically safe gravity is based on the idea of the dependence of the gravitational coupling upon the distance from the origin, approaching its classical value in the weak field regime. We consider three cases of identifying the cut-off parameter in the asymptotically safe gravity, leading to the three distinctive models for black holes. We find that the deviation of the fundamental mode from the Schwarzschild limit is a few percent, in contrast to the higher overtones, where the deviation reaches hundreds of percent, even when the fundamental mode almost coincides with the Schwarzschild mode. This behavior is connected with the fact that the quantum correction to the black hole spacetime is strong near the event horizon, but quickly falls off with distance and negligible near the peak of the effective potential surrounding the black hole.

# PROPERTIES AND RINGDOWN OF ROTATING BLACK HOLES IN EINSTEIN-SCALAR-GAUSS-BONNET GRAVITY

Jutta **KUNZ**

School of Mathematics and Science, Universität Oldenburg,  
Oldenburg 26129, Germany  
0000-0001-7990-8713  
jutta.kunz@uni-oldenburg.de

## Abstract

Testing the Kerr paradigm via current and future observations represents a major current theme in astrophysics and gravitational physics. From a theoretical point of view Einstein-scalar-Gauss-Bonnet theories are well motivated alternative gravity theories, that contain quadratic curvature terms and an additional scalar degree of freedom. For a dilatonic coupling function the resulting Einstein-dilaton-Gauss-Bonnet theory allows only for black holes with scalar hair. In contrast, for Einstein-scalar-Gauss-Bonnet theories with even coupling functions, there are scalarized and Kerr black holes. The domains of existence and further properties of the scalarized rotating black holes are discussed, as well as the quasinormal mode spectrum of the dilatonic scalarized rotating black holes. The presence of the scalar field breaks the isospectrality of the Kerr solutions. Moreover, close to the boundary of the domain of existence, the damping times of the modes vary considerably and the characteristics of the longest lived modes change.

## THE SOUND OF THE EVENT HORIZON

Roman **KONOPLYA**

Research Centre for Theoretical Physics and Astrophysics, Institute of Physics, Silesian University in Opava,  
Bezručovo náměstí 13, Opava, CZ-74601, Moravskoslezský kraj, Czech Republic  
0000-0003-1343-9584  
roman.konoplya@gmail.com

### Abstract

During the ringdown phase of a gravitational signal emitted by a black hole, the least damped quasinormal frequency dominates. If modifications to Einstein's theory induce noticeable deformations of the black-hole geometry only near the event horizon, the fundamental mode remains largely unaffected. However, even a small change near the event horizon can significantly impact the first few overtones, providing a means to probe the geometry of the event horizon. Overtones are stable against small deformations of spacetime at a distance from the black hole, allowing the event horizon to be distinguished from the surrounding environment. In contrast to echoes, overtones make a much larger energy contribution. These findings open up new avenues for future observations. In addition, in this talk we review recent works devoted to outburst of overtones in various theories of gravity.

# EXACT REGULAR BLACK HOLE SOLUTIONS WITH DE SITTER CORES AND ITS SHADOW CAST

Ali ÖVGÜN

Physics Department, Eastern Mediterranean University,  
Famagusta, 99628 North Cyprus, via Mersin 10, Türkiye  
0000-0002-9889-342X  
aliovgun@gmail.com

## Abstract

In this talk, I'll introduce a black hole solution that uses a variable equation of state,  $P = k(r)\rho$ , to describe both regular and singular black holes. The type of black hole depends on two key parameters,  $M_0$  and  $w_0$ . Regular black holes are especially interesting because they avoid the problem of singularities, which are points where physics breaks down. I'll also explore the shadow this black hole creates—essentially, the dark silhouette it casts against light from surrounding material—and show how we can use this shadow to understand and constrain the parameters of the solution. This helps us learn more about what's happening in the extreme environments near black holes [1] (Appears in CQG).

## References

- [1] V. Vertogradov, A.Övgün, arXiv: 2408.02699 [gr-qc].

## WHAT IF BLACK HOLES ARE SINGULARITY-FREE?

Piero **NICOLINI**

Dipartimento di Fisica, Università degli Studi di Trieste and  
Istituto Nazionale di Fisica Nucleare (INFN), Sezione di Trieste, Trieste, Italy  
Institut für Theoretische Physik, Johann Wolfgang Goethe-Universität and  
and Frankfurt Institute for Advanced Studies (FIAS), Frankfurt am Main, Germany  
0000-0001-7286-9774  
nicolini@fias.uni-frankfurt.de

### Abstract

Singularity theorems are important results in general relativity that leave little room for the fate of collapsing matter. Against this background, since the 1960s there have been many attempts to derive black hole solutions with a regular center. In this talk, I will discuss the main features of regular black holes and emphasize their surprising new phenomenology. Finally, I will draw some future scenarios of what can be learned from such spacetimes for the understanding of fundamental physics.

## QUANTUM SUPERPOSITIONS OF BLACK HOLES

Robert MANN

Department of Physics and Astronomy, University of Waterloo  
Waterloo, Ontario, N2L 3G1, Canada  
0000-0002-5859-2227  
rbmann@uwaterloo.ca

### Abstract

If relativistic gravitation has a quantum description, it must be meaningful to consider a spacetime metric in a genuine quantum superposition. But how might such a superposition be described, and how could observers detect it? I will present a new operational framework for studying “superpositions of spacetimes” via model particle detectors. After presenting the general approach, I show how it can be applied to describe a spacetime generated that is a superposition of two expanding spacetimes. I will then move on to show how black holes in two spatial dimensions can be placed in a superposition of masses and how such detectors would respond. The response exhibits signatures of quantum-gravitational effects reminiscent of Bekenstein’s seminal conjecture concerning the quantized mass spectrum of black holes in quantum gravity. I will provide further remarks concerning the meaning of the spacetime metric, and on distinguishing spacetime superpositions that are genuinely quantum-gravitational, notably with reference to recent proposals to test gravitationally-induced entanglement.

## RINGDOWN OF BLACK HOLES WITH INFINITE TOWER OF HIGHER-CURVATURE CORRECTIONS

Alexander **ZHIDENKO**

Centro de Matemática, Computação e Cognição (CMCC), Universidade Federal do ABC (UFABC)  
Rua Abolição, Santo André, CEP: 09210-180, SP, Brazil  
0000-0001-6838-3309  
olexandr.zhydenko@ufabc.edu.br

### Abstract

I will discuss the potential of using the quasinormal mode spectrum of black holes to probe higher-curvature corrections to General Relativity. Specifically, I will focus on a framework proposed by Bueno, Cano, and Hennigar [arXiv:2403.04827], which incorporates an infinite tower of higher-curvature corrections into Einstein's theory and allows for regular D-dimensional black holes. While the fundamental mode is minimally affected by moderate coupling constants, the higher overtones exhibit significant sensitivity even to small coupling values, yielding unconventional modes characterized by vanishing real oscillation frequencies. When comparing the frequencies derived from the metric truncated at several orders of higher-curvature corrections with those resulting from the infinite series of terms, we observe a rapid convergence of the frequencies to their limit for the complete regular black hole. This validates the extensive research conducted on specific theories with a finite number of higher-curvature corrections, such as the Lovelock theory. I will also consider the D-dimensional extension of the Dymnikova black hole, which is essentially nonperturbative in the coupling parameter that cannot be captured by theories with a finite number of curvature terms. Nevertheless, the associated modifications to the black-hole geometry can still be probed through the higher overtones of the quasinormal spectrum.

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## SCHWARZSCHILD BLACK HOLE IN GALAXIES SURROUNDED BY A DEHNEN-TYPE DARK MATTER HALO

Ahmad **AL-BADAWI**

Department of Physics, Al-Hussein Bin Talal University  
P. O. Box: 20, Ma'an 71111, Jordan  
ahmadbadawi@ahu.edu.jo

### Abstract

In this talk, we derive a novel Schwarzschild-like black hole (BH) solution describing a static and asymptotically flat BH surrounded by a dark matter (DM) halo with a Dehnen-type density distribution in the surrounding environment. We investigate the properties of the obtained BH by studying the curvature properties and energy conditions in Einstein gravity. Furthermore, we explore the features of a novel Schwarzschild-like BH embedded in a DM halo with Dehnen-type density profile by analyzing the timelike geodesics of particles along with BH observable properties.



## OBSERVATIONAL IMPRINTS OF SOLUTIONS FROM SMEARED SOURCES

Oleksandr **STASHKO**

Department of Physics, Princeton University  
Princeton, NJ, 08544, USA  
0000-0003-4734-9552  
alexander.stashko@gmail.com

### Abstract

We analyze in detail the possible observational properties of static, spherically symmetric solutions with smeared matter sources, including black holes and horizonless configurations. For a specific density distribution in the form of a Gaussian source, we study geodesic motion and the possible observational signatures of geometrically thin accretion disks, as well as the relativistic broadening of emission lines. We demonstrate that, while black hole cases are practically indistinguishable from the Schwarzschild black hole case, the horizonless configurations exhibit complex accretion disk structures and significant changes in the form of emission lines, which could lead to potentially observable differences. Additionally, we investigate the stability and quasinormal spectra of these configurations under axial gravitational perturbations. We show the appearance of significant peculiar features in the quasinormal mode spectra.

## SOME NEW PROPERTIES OF BLACK HOLES IN THE QUANTUM OPPENHEIMER-SNYDER MODEL

Farokhnaz **HOSSEINIFAR**

Department of Physics, Faculty of Science, University of Hradec Králové  
Rokitanského 62, 500 03 Hradec Králové, Czechia  
0009-0003-7057-451X  
f.hoseinifar94@gmail.com

Shi-Hai **DONG**

Centro de Investigación en Computación, Instituto Politécnico Nacional  
UPALM, CDMX 07700, Mexico  
0000-0002-0769-635X  
dongsh2@yahoo.com

Filip **STUDNICKA**

Department of Physics, Faculty of Science, University of Hradec Králové  
Rokitanského 62, 500 03 Hradec Králové, Czechia  
0000-0001-6721-8678  
filip.studnicka@uhk.cz

Hassan **HASSANABADI**

Department of Physics, Faculty of Science, University of Hradec Králové  
Rokitanského 62, 500 03 Hradec Králové, Czechia  
0000-0001-7487-6898  
hha1349@gmail.com

### Abstract

Due to the limitations of the classical Oppenheimer-Snyder model of black holes, Lewandowski et al. [Phys. Rev. Lett. 130, 101501 (2023)] introduced a new black hole which addresses the limitation of the classical model. This quantum model includes corrections that allow for a detailed analysis of features. We focus on the thermodynamic properties, quasi-normal modes utilizing the Rosen-Morse approximation method, and the topology of photon spheres and the thermodynamic topology of the black hole.

# INVESTIGATING THE WHITE DWARF RADIUS IN THE KAPPA DEFORMATION FORMALISM USING THE LANE-EMDEN EQUATION

Fariba **KAFIKANG**

Faculty of Physics, Shahrood University of Technology, Shahrood, Iran,  
0000-0001-7110-3830  
f.kafi19@yahoo.com

Hassan **HASSANABADI**

Faculty of Physics, Shahrood University of Technology, Shahrood, Iran,  
Department of Physics, University of Hradec Kralove  
Rokitanskeho 62/26, 500 03 Hradec Kralove, Czechia  
0000-0001-7487-6898  
hha1349@gmail.com

Won Song **CHUNG**

Department of Physics and Research Institute of Natural Science, College of Natural Science, Gyeongsang National University  
Jinju 660-701, Korea  
mimip44@naver.com

## Abstract

In this article, we solved the Lane-Emden equation, first, we calculated the pressure and potential due to gravity, and considering the density resulting from solving this equation, we obtained the radius of the white dwarf in different states. We obtained the ordinary cases when the deformation parameter goes to zero.

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## TESTING QUANTUM GRAVITY WITH DILUTE DIPOLAR BOSE GASES

Asma **TAHAR TAIBA**

Department of Physics, Faculty of Sciences, University of Blida 1  
Route SOUMAA, P.O. Box. 270, 09000, Blida, Algeria  
0009-0000-6220-5480  
tahartaibaasma5@gmail.com

Abdlelaali **BOUDJEMAA**

Department of Physics, Faculty of Exact Sciences and Informatics, Hassiba Benbouali University of Chlef  
P.O. Box 78, 02000, Chlef, Algeria  
0000-0002-5849-8310  
a.boudjemaa@univ-chlef.dz

### Abstract

This research aims to investigate the effects of quantum gravity on the ground-state properties of dilute homogeneous dipolar Bose gases. We utilize the Hartree-Fock-Bogoliubov theory combined with the Generalized Uncertainty Principle (GUP), incorporating both linear and quadratic terms in momentum, which implies a minimum measurable length parameterized by  $\alpha_0$  and  $\beta_0$ . We calculated quantum gravity corrections to the condensed fraction of bosons. The results demonstrate significant improvements in bounds on the GUP parameters compared to those set by high-energy physics and other experiments and theories. Therefore, the findings reveal that the interplay between dipole-dipole interactions (DDIs) and quantum gravity corrections enhances the bounds on the GUP parameters. This study represents significant progress toward improving bounds on the GUP parameters in dilute dipolar Bose gases, offering a potential for probing the reconciliation of gravity with quantum mechanics in current experimental setups.

## CHALLENGES IN COSMOLOGY: OBSERVING AND MODELING THE ACCELERATING UNIVERSE

Abdelhakim **BENSAID**

Mathematical Physics Research Team, Theoretical and Didactic Physics Laboratory, Faculty of Physics, USTHB  
 El Alia, Bab Ezzouar, B.P. 32, Alger, Algérie, 16111  
 0000-0003-3684-6537  
 abdelhakim.bensaid@usthb.edu.dz

### Abstract

This paper investigates the interplay between viscous Chaplygin gas and dark energy within the framework of modified gravity theory  $F(R,T)$ , where  $R$  represents the Ricci scalar and  $T$  denotes the trace of the energy-momentum tensor. The study is conducted in a Friedmann–Robertson–Walker (FRW) universe, where the dynamics are influenced by the combined contributions of Chaplygin gas and dark energy. We analyze how bulk viscosity impacts the dark energy equation of state parameter, focusing on two distinct interaction scenarios derived from the modified Friedmann equations. The results demonstrate that in both interaction models, the dark energy evolves into a quintessence-like phase, shedding light on its behavior in the presence of viscosity and interactions.

# ANALYTICAL SOLUTION OF THE KLEIN-GORDON EQUATION FOR NONPOLYNOMIAL ISOTONIC OSCILLATOR POTENTIAL TYPE USING A PROPER QUANTIZATION RULE

**Nadjat BENCHIHEUB**

Laboratory of Materials Physics, Radiation and Nanostructures, Faculty of Technology, University Mohamed El  
Bachir El Ibrahimi of Bordj Bou Arreridj  
Bordj Bou Arreridj, 34000, Algeria  
nadjat.benchiheub@univ-bba.dz

**N. GRAR**

Faculty of Technology, University Mohamed El Bachir El Ibrahimi of Bordj Bou Arreridj  
Bordj Bou Arreridj, 34000, Algeria.  
grar.nabila@univ-bba.dz

**N. BIOUD**

Laboratoire d'Optoélectronique et Composants, Université Ferhat Abbas de Sétif 1, Faculty of Technology,  
University Mohamed El Bachir El Ibrahimi of Bordj Bou Arreridj  
Bordj Bou Arreridj, 34000, Algeria  
nadhira.bioud@univ-bba.dz

**L. BAZIZ**

Abdelhafid Boussouf university center of Mila  
Mila, 43000, Algeria  
leila\_1b@yahoo.fr

**M. BERREHAIL**

Department of Physics, University of Constantine 1  
Constantine, 25000, Algeria  
Mounira.berrehail@umc.edu.dz

## Abstract

By exploiting the similarity existing between the non-relativistic Schrödinger equation and the relativistic Klein-Gordon equation, a proper quantization rule formulation in relativistic quantum mechanics can be established. We investigated this rule to solve the Klein-Gordon equation at one dimensional space and propose an exact solution for the isotonic oscillator potential type which is quasi exactly solvable, by assuming that the Scalar  $S(x)$  and Vector  $V(x)$  potentials are related by an algebraic relation. We calculate the exact relativistic energy levels for the considered potential by resolving the non-linear first order Riccati differential equation as well as the proper quantization rule. Our finding agree well with those obtained in the literature.

# 1 Introduction

This study presents an investigation to find exact solution of relativistic Klein-Gordon wave function using exactly methods of non-relativistic quantum mechanics. We are interested in the quantization methods which are based on the Wentzel-Kramer-Brillouin (WKB) approximation, especially the proper quantization rule [1]. This rule is derived from Ma, and Xu [2, 3] method, and ameliorated by Qiang and Dong [1]. Relative to this rule one can obtain results by solving the Klein-Gordon wave function which is reduced to a non-relativistic Schrödinger like equation. In the frame of the Qiang Dong rule an exact analytical solution for the relativistic energy level are found by resolving the Riccati non-linear first-order differential equation that is obtained when making a suitable transformation of the Schrödinger equation as well as the proper quantization rule. It becomes easier to solve the Riccati differential equation to find solution than to find it from the radial Schrödinger equation. The mixture of unequal scalar-vector potentials is also the subject of several works [4–8], where authors have taken the vectorial potential  $V(x)$  different from the scalar  $S(x)$  one and related by some algebraic relations; in fact, in some cases, they obtain approximate analytical bound state solutions. Our aim is to develop a relativistic Qiang-Dong proper quantization rule able to deal with the s-wave Klein-Gordon equation. This method is applied to re-calculate the relativistic energy spectrum for isotonic oscillator interaction type which involves a set of hyperbolic potentials. Notice that this family of potential [9] is quasi exactly solvable, and the determination of the relativistic energy spectrum  $E_n$  for non-linear potential is somewhat impossible and limited using the Qiang Dong quantization rule. This difficulty is overcome when the potential is decomposed by an exactly potential plus a term which is considered as a perturbative term [10].

# 2 Relativistic Proper quantization rule reformulation

The proper quantization rule, proposed by Qiang and Dong [1] of non-relativistic quantum mechanics, is more simplest in its calculation, and is given by the following equation (assuming that  $\hbar = m = 1$ ):

$$\int_{x_A}^{x_B} K_n(x)dx - \int_{x_{0A}}^{x_{0B}} K_0(x)dx = n\pi \quad (1)$$

with  $K_n(x) = \sqrt{2m[E_n - V(x)]}$ , with  $E_n \geq V(x)$  is the classical momentum function for the energy  $E_n$  between two classical turning points  $x_A$  and  $x_B$ . A spatially one-dimensional, time-independent Klein-Gordon [11–14] equation, for a spinless particle of rest mass  $m$  in the presence of mixed vector  $V(x)$  and Scalar potentials  $S(x)$  is written as:

$$\frac{-d^2\psi(x)}{dx^2} + 2m[(m + S(x))^2 - (E_n - V(x))^2]\psi(x) = 0 \quad (2)$$

When,  $V(x)$  is the time-component called Lorentz vector potential, and  $S(x)$  is the space-time scalar potential. while keeping the constant of Planck  $\hbar = 1$  and the velocity of light  $c = 1$  throughout the rest of this work.  $E_n$  denotes the total relativistic energy of a spinless particle for a bound state  $n$ . Which can be rewritten as a Schrödinger equation for unequally mixed potentials  $V(x)$  and  $S(x)$  will be expressed by:

$$\frac{d^2\psi(x)}{dx^2} + 2m[E_{(eff,n)} - V_{eff}(x)]\psi(x) = 0 \quad (3)$$

hence,  $E_{(eff,n)} = \frac{(E_n^2 - m^2)}{2m}$  and  $V_{eff}(x) = \frac{[V^2 - S^2 - 2(E_n V(x) + mS(x))]}{2m}$  denote respectively the effective energy of a particle moving in an effective potential. When performing the transformation,  $\phi_0(x) = \frac{1}{\psi(x)} \frac{d\psi(x)}{dx}$  which refers to the logarithmic derivative, the Schrödinger equation in 3 is reduced to the nonlinear differential equation of the first order of Riccati equation for the ground state and becomes;

$$\frac{d\phi_0}{dx} = -2m[E_{(eff,0)} - V_{eff}(x)] - \phi_0^2 \quad (4)$$



Based upon the fact that the Equations of (2) and (3) are equivalent, we can deduce that the proper quantization rule for the relativistic Klein-Gordon is identical to the nonrelativistic proper quantization rule, and will be expressed as:

$$\int_{x_A}^{x_B} K_n(x)dx - \int_{x_{0A}}^{x_{0B}} K_0(x)dx = n\pi \quad (5)$$

with  $k_n = \sqrt{2m[E_{(eff,n)} - V_{eff}(x)]}$ , in the region of  $E_{(eff,n)} \geq V_{eff}(x)$  is analogous to classical momentum between  $x_A < x < x_B$ , since  $x_A$  and  $x_B$  are the two turning points ( $x_A < x_B$ ), and can be determined by calculating  $E_{(eff,n)} = V_{eff}(x)$ , when  $V_{eff}(x)$  is the effective solvable potential.

### 3 Energy spectrum of the isotonic oscillator potential

Let us in this section apply the relativistic Qiang Dong proper quantization Eq. (5) in order to find the bound states of bosons by resolving the relativistic Klein-Gordon wave function for the isotonic oscillator type involving a set of a family of interaction potentials such as: Pöschl-Teller, Pöschl-Teller II, Scarf II potentials. Which takes a nonlinear form similar to the harmonic oscillator plus a centrifugal barrier considered as a small perturbation in the inverse-square singular form 9:

$$V_{inter}(y) = \lambda_2 y^2 + \frac{\mu_2}{y^2} + \lambda_0 \quad (6)$$

Where  $\lambda_2$ ,  $\mu_2$ , and  $\lambda_0$  are positive constants. The derivative of  $y$  with respect to  $x$  satisfies to  $\frac{dy}{dx} = \alpha(1 - y^2) > 0$ . Knowing that the variable  $y$  is function of  $x$ . To calculate the relativistic energy eigenvalues, we supposed that the Vector and the Scalar potentials are constrained by the relation:  $V(x) = -S(x) + \beta$ , where  $\beta$  is a constant. Therefore, the scalar potential function has a nonlinear form such as:  $S(x) = S_0(\varepsilon y^2 + \frac{\gamma}{y^2} + C)$ , when  $\varepsilon$ ,  $\gamma$ , and  $C$  are positive constants. In the other hand, the effective potential depends upon the relativistic energy of the particle, and is expressed in terms of Scalar potential by:

$$V_{eff}(x) = \frac{2(-\beta + E_n + m)S(x) + (2E_n\beta - \beta^2)}{2m} \quad (7)$$

Furthermore, we suggest that the coefficients of the interaction model have the formulations:  $\lambda_2 = [2(-\beta + E_n + m)\varepsilon S_0]$ ,  $\mu_2 = [2(-\beta + E_n + m)S_0\gamma]$ , and  $\lambda_0 = [2(-\beta + E_n + m)S_0C + (2E_n\beta - \beta^2)]$ . Perform the variable changing where  $y^2 = z$ , the two turning points  $z_A, z_B$  with ( $z_A < z_B$ ) can be obtained by solving the second-order polynomial obtained from the equality  $E_{(eff,n)} = V_{eff}(x)$ ,

$$z_A = \frac{-1}{2} \left[ \frac{C}{\varepsilon} + \frac{(2E_n\beta - \beta^2) - 2mE_{(eff,n)}}{2(-\beta + E_n + m)\varepsilon S_0} \right] - \frac{1}{2} \sqrt{\Delta} \quad (8)$$

$$z_B = \frac{-1}{2} \left[ \frac{C}{\varepsilon} + \frac{(2E_n\beta - \beta^2) - 2mE_{(eff,n)}}{2(-\beta + E_n + m)\varepsilon S_0} \right] + \frac{1}{2} \sqrt{\Delta} \quad (9)$$

$\sqrt{\Delta} = \sqrt{\left[ \frac{C}{\varepsilon} + \frac{(2E_n\beta - \beta^2) - 2mE_{(eff,n)}}{2(-\beta + E_n + m)} \right]^2 - 4\frac{\gamma}{\varepsilon}}$ , is the root of discriminant of the second order polynomial. Proceeding now the ground energy determination, find from the resolution of the mathematical Riccati equation Eq.(4) without having need to solve the Schrödinger equation: Where the logarithmic derivative  $\phi_0 = -ay(x) + \frac{b}{y}$  satisfies the above Riccati differential nonlinear equation (4). Resolving the equation systems obtained from the last equation,

we can find the unknown coefficients  $a$  and  $b$  of the Riccati function and the fundamental energy given as :

$$a = \left(-\frac{\alpha}{2}\right) + \frac{\alpha}{2} \sqrt{1 + \frac{8(-\beta + E_n + m)\varepsilon S_0}{\alpha^2}} \quad (10a)$$

$$b = \frac{\alpha}{2} \pm \frac{\alpha}{2} \sqrt{1 + \frac{8(-\beta + E_n + m)\gamma S_0}{\alpha^2}} \quad (10b)$$

$$E_{(eff,0)} = \frac{1}{2m} [-(a-b)^2 + 2S_0(\gamma + \varepsilon + C)(-\beta + E_n + m) + (2E_n\beta - \beta^2)] \quad (10c)$$

Now the eigenvalues energies are found from the proper quantization rule after calculating the first integral momentum between the turning points  $z_A(x_A), z_B(x_B)$  in the region of  $E_{(eff,n)} \geq V_{eff}$ ;

$$\int_{x_A}^{x_B} K_n(x) dx = \int_{x_A}^{x_B} \sqrt{2m(E_{(eff,n)} - V_{eff})} dx \quad (11)$$

$$= \frac{\pi \sqrt{(2 - \beta + E_n + m)\varepsilon S_0}}{2\alpha} \left[ 1 - \frac{C + \gamma}{\varepsilon} + \frac{(2E_n\beta - \beta^2) - 2mE_{(eff,n)}}{2(-\beta + E_n + m)\varepsilon S_0} - \sqrt{\frac{\gamma}{\varepsilon}} \right] \quad (12)$$

in the other hand, the second integral  $\int_{x_{0A}}^{x_{0B}} K_0 dx$  is deduced directly from the first integral, by replacing the energy  $E_{(eff,n)}$  in the last equation with the ground state energy expression  $E_{(eff,0)}$  given in Eq.(10) and considering the relativistic proper quantization rule Eq.(5), the exact explicit form of the eigenvalues energy spectrum for the isotonic oscillator has the closed form

$$E_n^2 - m^2 = -(2n\alpha - a + b)^2 + 2S_0(\gamma + \varepsilon + C)(-\beta + E_n + m) + (2E_n\beta - \beta^2). \quad (13)$$

In another way in terms of the coefficient of the isotonic oscillator interaction potential, the The closed form of the relativistic energy is given by:

$$E_n^2 - m^2 = -(2n\alpha - a + b)^2 + (\lambda_2 + \mu_2 + \lambda_0) \quad (14)$$

Hence,  $E_n^+$  and  $E_n^-$  refers to the energy of the particle and the antiparticle respectively. Since, when the zero constant  $\beta$  is taken, we return to the case of a mixture equally of scalar and vector potentials, thus, the analytical expression of the relativistic energy find is;

$$E_n^2 - m^2 = -(2n\alpha - a + b)^2 + 2S_0(\gamma + \varepsilon + C)(E_n + m) \quad (15)$$

Thus, the energy spectrum obtained retains its property of exactness despite the presence of the perturbative term; this latter seems to have no effect on the exactness of the result. Else the energy spectrum of the isotonic oscillator potential is related to this of the radial harmonic oscillator shifted by an amount constant.

## 4 Conclusion

In this work, we have found that the K.G wave function can be represented as a nonrelativistic Schrödinger equation. This way it is possible to find a proper quantization rule for the relativistic Klein-Gordon equation, in one dimensional space and is expressed as:  $\int_{x_A}^{x_B} K_n(x) dx - \int_{x_{0A}}^{x_{0B}} K_0(x) dx = n\pi$ , with  $k_n = \sqrt{2m[E_{(eff,n)} - V_{eff}(x)]}$  analogous to classical momentum,  $x_A$  and  $x_B$  are the two turning points ( $x_A < x_B$ ),  $V_{eff}(x)$  is the effective solvable potential,  $E_{(eff,n)}$  is the effective energy and  $n$  is a quantum number and represents the number of the nodes of the wave function  $\psi(x)$ . The isotonic oscillator potential, which have an extension to the harmonic oscillator, considering, that these extension represent a perturbative terms, an exact analytical expression of the energy levels is obtained, which means that the perturbative term has no effect on the energy spectrum. As we can see, this rule is very useful to provide eigenvalues for exactly solvable potential both in relativistic and nonrelativistic quantum mechanics with simple and easy calculations of the Riccati first order differential equation without ever solving the Schrödinger equation.

## 4.1 Appendix

During the calculations of the momentum  $K(x)$  integral, we have used the following integrals formula 15–18

$$\int_{z_A}^{z_B} \frac{dz}{\sqrt{(z - z_A)(z_B - z)}} = \pi, (z_A < z_B) \quad (16a)$$

$$\int_{z_A}^{z_B} \frac{dz}{(cz + d)\sqrt{(z - z_A)(z_B - z)}} = \frac{\pi}{\sqrt{(d + cz_A)(d + z_B)}}, (z_A < z_B, c \neq 0) \quad (16b)$$

$$\int_{z_A}^{z_B} \frac{\sqrt{(z - z_A)(z_B - z)} dz}{z(1 - z)} = \pi[-\sqrt{z_A z_B} + 1 - \sqrt{(1 - z_A)(1 - z_B)}] \quad (16c)$$

$$\int_{z_A}^{z_B} \frac{\sqrt{(z - z_A)(z_B - z)} dz}{(1 - z^2)} = \frac{\pi}{2}[2 - \sqrt{(z_A - z_B)} - \sqrt{(1 + z_A)(1 + z_B)}], (-1 < z_A < z_B < +1) \quad (16d)$$

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## SYNCHROTRON RADIATION AND SELF-ABSORPTION EFFECTS IN GAMMA RAY BURST AFTERGLOWS

Esmā **ZOUAOUI**

Laboratoire de Physique Mathématique et Subatomique (LPMPS), Physics Department, Faculty of Fundamental  
Sciences, Constantine 1 Frères Mentouri University  
Constantine, Algeria  
0009-0003-0000-8002  
esma.zouaoui@gmail.com

Noureddine **MEBRAKI**

Department or Lab, Faculty, University  
Laboratoire de Physique Mathématique et Subatomique (LPMPS), Physics Department, Faculty of Fundamental  
Sciences, Constantine 1 Frères Mentouri University  
Constantine, Algeria  
*Unfortunately he passed away*

### Abstract

After the big bang, the Gamma-ray bursts (GRBs) are the most energetic phenomena observed to date. In this work we aim to explain the emission happened along this phenomenon, focusing on the interaction of the jet with the surrounding environment, referred to as GRB afterglows. We attempt to address the radiation of GRB afterglows through synchrotron emission mechanisms and study the limit of synchrotron self-absorption. The diffusion of Inverse Compton scattering is negligible.

## BIOFILM FORMATION ON API 5L X52 STEEL SURFACES USED IN ALGERIAN CRUDE OIL PIPELINES AND ITS MITIGATION STRATEGIES

Hadjer **DIDOUH**

Process Engineering Department, Laboratory of Theoretical Physics and Materials Physics, Faculty of Technology,  
University of Chlef, Chlef, Algeria  
0000-0001-8387-0869  
h.didouh@univ-chlef.dz

Mohammed **HADJ MELLIANI**

Mechanical Engineering Department, Laboratory of Theoretical Physics and Materials Physics, Faculty of  
Technology, University of Chlef, Chlef, Algeria  
0000-0003-1375-762X  
m.hadjmeliani@univ-chlef.dz

Izzeddine **SAMEUT BOUHAİK**

Process Engineering Department, Laboratory of Theoretical Physics and Materials Physics, Faculty of Technology,  
University of Chlef, Chlef, Algeria  
0000-0002-1987-6569  
i.sameutbouhaik@univ-chlef.dz

### Abstract

The vast network of crude oil pipelines traversing Algeria faces a hidden but significant threat—biofilm-induced corrosion. Our study focuses on unraveling the mechanisms behind biofilm adhesion and biocorrosion on API 5L X52 carbon steel surfaces, commonly used in Algerian oil and gas pipelines. This research combines surface characterization, electrochemical analysis, and microbial assays to explore how biofilm-forming microorganisms interact with steel surfaces, particularly in the presence of sulfate-reducing bacteria (SRB). The findings reveal that surface topography and hydrophilicity play critical roles in biofilm attachment, with increased surface roughness facilitating stronger adhesion. Through scanning electron microscopy (SEM) and energy-dispersive X-ray spectroscopy (EDS), we detected iron sulfide deposits within pitted corrosion sites, confirming microbial corrosion activity. Gravimetric analysis showed significant pitting corrosion after 21 days of exposure, with corrosion rates escalating over time in simulated production water environments. Additionally, the study investigates potential mitigation strategies. *Moringa Oleifera*, used as a biocide,

demonstrated promise in reducing biofilm formation, while surface energy modification of the steel reduced microbial attachment. This research underscores the importance of addressing microbial adhesion and corrosion in crude oil pipelines, offering a framework for preventing the long-term deterioration of critical infrastructure.

# GPVDM SIMULATION OF LAYER THICKNESS EFFECT AND WORK TEMPERATURE ON POWER CONVERSION EFFICIENCY OF $CH_3NH_3PbI_3$ BASED PEROVSKITE SOLAR CELL

Ourida **OURAHMOUN**

Electronic Department, LATAGE laboratory, Faculty of Electrical and Computer Engineering, Mouloud Mammeri University (UMMTO),  
BP 17 RP 15000, Tizi-Ouzou, Algeria  
0009-0008-2705-5315  
ourida.ourahmoun@ummto.dz

## Abstract

Perovskite-based solar cell technologies have been a very attractive area of research in recent years. Organic-inorganic perovskite materials are in an increased evolution in power conversion efficiency. In this paper, we used the GPVDM software to study the effect of some parameters on power conversion efficiency in a planar heterojunction solar cell based on  $CH_3NH_3PbI_3$  as an absorbing layer. The structure of the cell is  $FTO/TiO_2/CH_3NH_3PbI_3/SpiroOMeTAD/Au$ .  $TiO_2$  used as an electron transport layer (ETL), Spiro-OmeTAD is used as a hole transport layer (HTL), and  $MAPbI_3$  is used as an active absorbing layer. Organic and inorganic materials are used as buffer layers to facilitate charge transport in the photovoltaic cell. The results show that the efficiency of the cells can be improved by adjusting the layer thickness of the different layers and the choice of the buffer layers. Power conversion efficiency was increased from 9% to 22%. The work temperature also affects the performance of the cell. The efficiency decreases when the temperature increases.

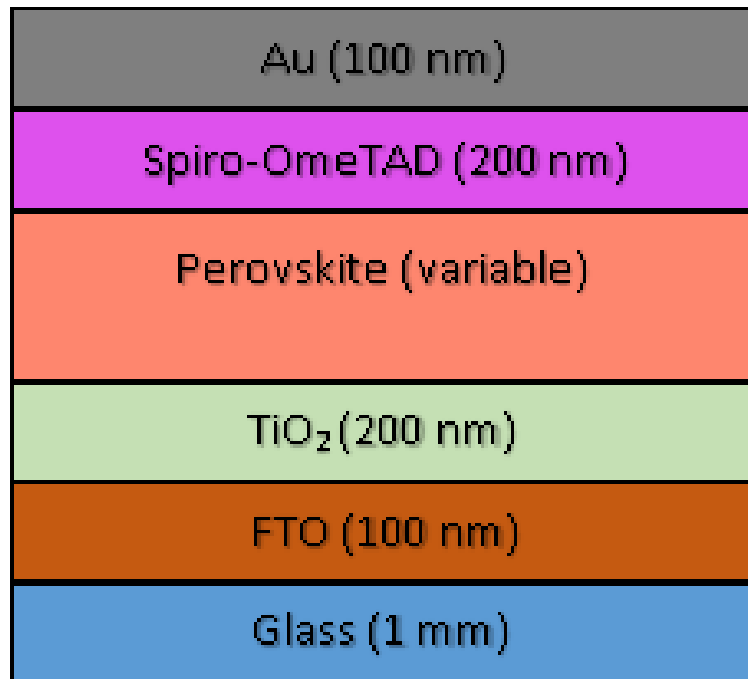


Figure 1: Structure of the solar cell.

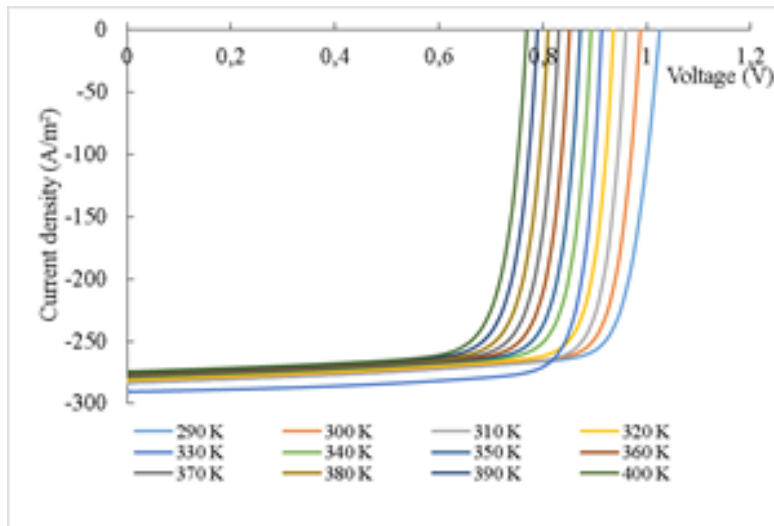


Figure 2: The current density as a function of temperature.



# ANDERSON LOCALIZATION OF BQPs OF QUANTUM LIQUIDS

Zohra **MEHRI**

Department of Physics, Faculty of Sciences and Technology, Ahmed Zabana University of Relizane  
Bourmadia, BP 48000, Relizane, Algeria  
0009-0006-3643-9623  
zohra.mehri@univ-relizane.dz

Abdelaali **BOUDJEMAA**

Department of Physics, Faculty of Exact Sciences and Informatics, Hassiba Benbouali University of Chlef  
P.O. Box 78, 02000, Ouled-Fares, Chlef, Algeria.  
0000-0002-5849-8310  
a.boudjemaa@univ-chlef.dz

## Abstract

In this study we investigate, both analytically and numerically, the Anderson localization of quasiparticles in quantum droplets subjected to correlated disordered potentials. Specifically, we calculate the density profiles, the Bogoliubov quasiparticle modes, and the localization length. Our findings reveal that in one-dimensional speckle potentials, the Anderson localization of quasiparticles in the droplet state is weak in the flat-top region, as the excitation modes are limited to those below the particle-emission threshold.

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# BLACK HOLE PHASE TRANSITION AND SIMILARITY BETWEEN NON-COMMUTATIVE SCHWARZSCHILD BLACK HOLE AND ADS-REISSNER-NORDSTRÖM BLACK HOLE

Abdellah **TOUATI**

Department of Physics, Faculty Science and Applied Science, University of Bouira  
10000-bouira, Bouira, Algeria  
0000-0003-4478-2529  
touati.abph@gmail.com

Slimane **ZAIM**

Department of Physics, Faculty of Sciences of Matter, University of Batna-1  
05000-batna, Batna, Algeria  
0000-0003-1221-923X  
zaim69slimane@yahoo.com

## Abstract

In this work, we investigate the phase transition of Schwarzschild black hole (SBH) in non-commutative (NC) gauge theory of gravity with a new scenario. First, we study the SBH thermodynamic in the grand canonical ensemble, as the electric potential in the Reissner-Nordström (RN) BH. Our result shows an important similarity between the NC SBH and the AdS RN one, where we identified a direct relation between the NC parameter and the electric charge given by  $\Theta^2 = \frac{4}{3}Q^2$ . Then, we present a new scenario in which the non-commutativity effect is treated as an external pressure applied by spacetime on the SBH, and that shows another new similarity between the effect of non-commutativity and the cosmological constant. Our result show two-coexistence and second-order phase transition in this geometry and a predicted a fundamental length at Planck scale  $\Theta \sim l_p$ , using this estimation one can obtain the constraint on the electric charge of the RN BH.

## A SCALE INVARIANT GEORGI MACHACEK MODEL

Hamza **TAIBI**

Department of Physics, University of M'Hamed Bougara Boumerdes  
35 000 Boumerdès, Algeria  
0000-0003-1443-1402  
htaibi87@gmail.com

### Abstract

In this work, we construct a scale-invariant extension of the Georgi-Machacek model [1], which is a triplet extension of the Standard Model (SM) that preserves custodial symmetry. Scale-invariant extensions of the SM were first introduced in the seminal paper by Coleman and Weinberg [2] and later extended to more complicated models involving multiple scalar fields by Gildner and Weinberg [3]. In our model, after writing the scalar potential of the extended Georgi-Machacek model, we use the Gildner method to find a minimum ground state called a flat direction. We then compute the one-loop effective potential and derive the mass spectrum of the model. Next, we apply theoretical constraints such as vacuum stability and perturbative unitarity, in addition to experimental constraints from the electroweak oblique parameter  $S$  and Higgs signal strength, to study the parameter space of the model. We find that there is a region in the parameter space of the model compatible with all constraints.

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## VAN DER WAALS BLACK HOLE IN DEFORMED SPACE WITH MAXIMAL LENGTH

**Rabah OUBAGHA**

L.S.D.C, Université de Oum-El-Bouaghi,  
Faculté des Sciences Exactes et SNV, Algeria  
0009-0008-5877-7142  
oubagha.rabah@gmail.com

**Bilel HAMIL**

Laboratoire de Physique Mathématique et Subatomique,  
Faculté des Sciences Exactes, Université Constantine 1 Frères Mentouri, Constantine, Algeria.  
0000-0002-7043-6104  
bilelhamil@gmail.com

**Bekir Can LUTFUOGLU**

Department of Physics, Faculty of Science, University of Hradec Kralove,  
Rokitanskeho 62/26, Hradec Kralove, 500 03, Czech Republic.  
0000-0001-6467-5005

**Mahmoud MERAD**

L.S.D.C, Université de Oum-El-Bouaghi,  
Faculté des Sciences Exactes et SNV, Algeria  
0000-0001-7547-6933  
meradm@gmail.com

### Abstract

In the context of the Generalized Heisenberg Uncertainty Principle (GUP), consistent with a maximum observable length scale, we propose studying the energy conditions underlying the GUP-corrected solutions to the Einstein equation for the Van Der Waals black hole. This is done by determining the GUP-corrected Hawking temperature associated with these solutions. Our results indicate that this formalism necessitates a restriction on the radius of the event horizon.

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## A LIGHTWEIGHT AND EFFICIENT SEMI-QUANTUM SECRET SHARING PROTOCOL

Mustapha Anis **YOUNES**

Université de Bejaia, Faculté des Sciences Exactes, Laboratoire de Physique Théorique,  
06000 Bejaia, Algérie  
0009-0007-4979-9887  
mustaphaanis.younes@univ-bejaia.dz

Sofia **ZEBBOUDJ**

ENSIBS, Université Bretagne Sud,  
56000, Vannes, France  
0000-0002-5721-5753  
sofia.zebboudj@univ-ubs.fr

Abdelhakim **Gharbi**

Université de Bejaia, Faculté des Sciences Exactes, Laboratoire de Physique Théorique,  
06000 Bejaia, Algérie  
0000-0003-2995-3238  
abdelhakim.gharbi@univ-bejaia.dz

### Abstract

Quantum secret sharing (QSS) [1] is a procedure that allows a dealer, namely as Alice, to distribute a secret among several participants in such a way that no individual participant has enough information to deduce the secret. The secret can only be reconstructed when a sufficient number of participants collaborate. Unlike classical methods [2], which rely on mathematical proprieties, the security of the shared secret is based upon the principles of quantum physics.

Since its introduction by Hillery et al. [1] in 1999, numerous QSS protocols have been developed [3–6], utilizing a variety of quantum resources. However, those protocols assume that all participants can possess full quantum capabilities. This assumption is unrealistic for the simple reason that certain quantum capabilities are expensive and challenging to implement using current technologies. Consequently, the critical challenge has become [7]: how can we achieve the objectives of quantum communication protocols while simultaneously minimizing the quantum overhead for participants?

This challenge was first addressed by Boyer et al. [8] in 2007 by introducing the concept of semi-quantum environment. This environment enables a fully quantum

user to securely share a secret with classical participants equipped with a limited set of quantum devices. While semi-quantum protocols are generally more practical than fully quantum ones, most of the proposed protocols typically employ a two-way or circular communication structure. These setups necessitate that classical participants to always be equipped with expensive quantum devices, such as photon number splitters (PNS) and wavelength filters (WF), to defend against quantum Trojan horse attacks. However, the requirement for such devices contradicts the fundamental purpose of semi-quantum environments, which is to provide low-cost protocols with restricted quantum capabilities, while maintaining security levels comparable to fully quantum ones.

In our recent work [9], we developed a novel, lightweight multi-party SQSS scheme in which the classical participants are restricted to performing only two operations: (1) measuring qubits in the  $Z$  basis, and (2) applying the Hadamard gate on single qubits. In this new scheme, Alice uses  $M$ -particle entangled states to share her secret and leverage the propriety between Bell states and the Hadamard gate to effectively detect eavesdroppers. This new protocol holds several merits and advantages compared to previous approaches: (a) The scheme adopts a fully one-way communication structure making robust against the Trojan horse attacks without requiring classical participants to use costly quantum devices; effectively aligning with the original intent of the semi-quantum environment. (b) The protocol offers superior qubit efficiency compared to other multi-party SQSS protocols. (c) It offers higher security against well-known attacks. (d) It offers higher flexibility as Alice can control the content of the secret she shares.

In the following presentation, we will highlight the key features of our protocol and illustrate how it can effectively address some of the issues encountered in semi-quantum secret sharing.

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## THE SOLITONIC SOLUTIONS OF HIROTA EQUATION

Zeyneb **TAIBI**

Laboratory for Theoretical Physics and Material Physics Faculty of Exact Sciences and Informatics, Hassiba  
Benbouali University of Chlef  
Oulad Fares, Chlef, 02000, Algeria  
0009-0009-4371-1439  
z.taibi@univ-chlef.dz

Houria **CHAACHOUA SAMEUT**

Laboratory for Theoretical Physics and Material Physics Faculty of Exact Sciences and Informatics, Hassiba  
Benbouali University of Chlef  
Oulad Fares, Chlef, 02000, Algeria  
0009-0000-4387-4048  
chachou.houria@yahoo.fr

### Abstract

A vast range of physical events can now be understood through the analytical study of nonlinear equations utilizing sophisticated mathematical methods like Lax pair and Darboux transformations. With an emphasis on clarifying soliton solutions, this work investigates the application of these techniques to the nonlinear Schrödinger equation, a classic nonlinear problem. Solitons are essential in many domains, such as hydrodynamics, quantum gases, optical fiber communication, and plasma physics, due to their stable, localized wave profiles. We prove the inerrability condition by using Darboux transformations and a lax pair to discover solutions that are analytically exactly as soliton. The impact of the coefficients in the soliton solution is then covered.



# BREATHERS SOLUTIONS OF NONLINEAR HIROTA EQUATION WITH HIGHER ORDER CUBIC NONLINEARITIES, HIGHER-ORDER DISPERSION, DAMPING AND EXTERNAL POTENTIAL

Houria **CHAACHOUA SAMEUT**

Laboratory for Theoretical Physics and Material Physics Faculty of Exact Sciences and Informatics, Hassiba  
Benbouali University of Chlef  
Oulad Fares, Chlef, 02000, Algeria  
0009-0000-4387-4048  
chachou.houria@yahoo.fr

Zeyneb **TAIBI**

Laboratory for Theoretical Physics and Material Physics Faculty of Exact Sciences and Informatics, Hassiba  
Benbouali University of Chlef  
Oulad Fares, Chlef, 02000, Algeria  
0009-0009-4371-1439  
z.taibi@univ-chlef.dz

Faiza **DEBBI**

M.Sc. Student in Hassiba Benbouali University of Chlef  
Oulad Fares, Chlef, 02000, Algeria

## Abstract

In recent decades, nonlinear equations have appeared in various forms to study the behavior of complex natural phenomena in different branches of science and technology. Most nonlinear phenomena are modelled in terms of a nonlinear evolutionary equation Hirota equation (HE) due to linear and nonlinear effects. The solution of these HE leads to solitary waves and periodic solutions, which plays an important role in the description of nonlinear physical phenomena. In this work we used the lax pair and Darboux transformation to solve the higher order hirota equation with time-dependent coefficients. We investigate the integrability of HE with damping and linear potential and find that it is integrable only with two independent coefficients, namely, the third-order dispersion  $a_3(t)$  and The selfsteepening  $a_4(t)$ . From our finding, we deduce that the passage from general to Akhmediev Breather controlled by the coefficient of self-steepening and the third-order dispersion can control the center of mass motion of the general breather in the optical fiber without an external potential.

# THE DUFFIN-KEMMER-PETIAU OSCILLATOR AND THE GENERALIZED MOMENTUM OPERATOR

M'hamed **HADJ MOUSSA**

Theoretical Physics and Radiation Matter Interaction Laboratory (LPThIRM),  
Physics Department, Sciences Faculty, University of Saad Dahlab - Blida 1  
PO box 270 Soumaa Road , 09000 , Blida, Algeria.  
0000-0003-3987-6790  
hadj2009@gmail.com

## Abstract

In this study, we solve the Duffin-Kemmer-Petiau oscillator equation within the framework of the displacement operator in one dimension by introducing a generalized momentum operator  $\hat{p}$ . This approach allows for the determination of the energy spectrum  $E_n$ , and we examine certain limits of the energy spectrum. Finally, we compute the corresponding wave functions  $\phi_i(x)$ , which are expressed in terms of the generalized Laguerre polynomials  $L_n^k(x)$ .

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## MOVEMENT OF BOSON PARTICLES WITH DIFFERENT TYPES OF POTENTIALS IN THE DEFAULT TOPOLOGICAL SPACETIME

Moussa **ABBAD**

Laboratory of Applied and Theoretical Physics, Echahid Cheikh Larbi-Tébessi University

Tebessa, Algeria

0009-0004-1626-9669

moussa.abbad@univ-tebessa.dz

Houcine **AOUNNALLAH**

Department of Science and Technology, Echahid Cheikh Larbi-Tébessi University

Tebessa, Algeria

0000-0002-0611-4063

houcine12400@gmail.com

### Abstract

In this presentation, we analyze scalar bosons interacting with vector potentials within the framework of a cosmic string spacetime. Using the DKP equation for spin-0 particles, we introduce a mass transformation of the form  $m \rightarrow (m + S(r))$  for various types of potentials. This approach leads to a second-order differential equation, which is identified as the hypergeometric equation.

To solve this differential equation, we apply the Frobenius method, expanding the solution as a power series around the origin. Through this process, we determine the energy levels and corresponding wave functions

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## ILLUSTRATIONS OF THE POST-GAUSSIAN APPROXIMATION ON SINGLE SPECIES

Fatima **KOUDJIL**

Laboratory for Theoretical Physics and Material Physics LPTPM QUANTUM GASES GROUP,  
Department of Physics, Faculty of Exact Sciences and Informatics, Hassiba Ben Bouali University  
Zeboudja 2014, Chlef, Algeria  
0009-0000-2169-9883  
fa.koudjil@univ-chlef.dz

Mohamed **BENAROUS**

Laboratory for Theoretical Physics and Material Physics LPTPM QUANTUM GASES GROUP, Department of  
Physics Faculty of Exact Sciences and Informatics Hassiba Ben Bouali University  
2000, Chlef, Algeria  
0009-0008-5957-4128  
m.benarous@univ-chlef.dz

### Abstract

In our work, we will present some initial illustrations of the post-Gaussian approximation for single Bose-Einstein condensates at zero temperature, focusing on the effects associated with triplets. The emphasis will be on determining the static and dynamic properties of a system containing a large number of particles using the Thomas-Fermi approximation.

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## EVALUATION OF THE ELECTRIC QUADRUPOLE MOMENT MICROSCOPIC, IN THE $58 < N < 82$ REGION, XENON

**Hadj MOULOUDJ**

Laboratoire de Physique des Particules et Physique Statistique,  
Ecole Normale Supérieure-Kouba, B.P. 92, Vieux-Kouba 16050, Algiers, Algeria  
Department of Physics, Faculty of Exact Sciences and Informatics, Hassiba Ben Bouali University,  
BP 151, Chlef 02180, Algeria  
0009-0006-0376-9642  
hadj.mouloudj@gmail.com

**Fethi REDJEM**

Department of Material Sciences, Faculty of Science, 35 G Gherdaia  
road -03000, Amar Telidji University, Laghouat, Algeria  
0000-0002-2937-9144  
f.redjem@lagh-univ.dz

**Youcef BELGAID**

Department of common core in Exact Sciences and Informatics,  
Hassiba Benbouali University, Chlef, Algeria  
0009-0008-5548-0140  
youcefb079@gmail.com

**Oussama ZEGGAI**

Research Unit Materials and Renewable Energy (URMER),  
Abou Bekr Belkaïd University, BP 119, Tlemcen, Algeria  
0000-0002-9850-3559  
zeggai-oussama@yahoo.com

**Alla Eddine TOUBAL MAAMAR**

University of M'hamed Bougara of Boumerdes,  
Boumerdes 35000, Algeria  
0000-0002-0738-0048  
allaeddine140dz@gmail.com

### Abstract

The deformation energy of some evens nucleus in the Xenon region Xe is studied by the Macroscopic-Microscopic method using the Strutinsky method [1], with shell and BCS of pairing corrections. The macroscopic energy is calculated from the liquid drop model. The shell corrections  $\delta E_{1,2}$  ( $N$  or  $Z$ ) [2] are extracted from a Schrödinger equation of Hartree-Fock-Skyrme type in which the self-consistent potential is replaced

by a deformed Woods-Saxon mean field [3]. From equilibrium shapes, we use single wave functions and BCS's approximation for deducing the electric quadrupole moments  $Q_{20}$  for the considered region. Good results are thus obtained especially the theoretical values are found quite close to the experimental ones. The numerical resolution of the equations is done using a FORTRAN program. The theoretical values are quite close to the experimental values.

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# DUAL VECTORS FOR HAMILTONIAN SYSTEMS: TOWARDS A UNIFIED REPRESENTATION

Ferhat TAŞ

Department of Computer Science, Faculty of Science, İstanbul University  
 Vezneciler Cad. 34134, İstanbul, Türkiye  
 0000-0001-5903-2881  
 tasf@istanbul.edu.tr

## Abstract

Hamiltonian systems provide a unified framework for understanding physical dynamics, leveraging both kinetic and potential energy functions. This paper explores the integration of dual vectors within Hamiltonian systems to offer a compact representation that combines position, momentum, and their derivatives. By linking dual vectors to symplectic geometry, this work introduces a novel mathematical approach that enhances the representation and analysis of physical systems.

## 1 Introduction

Hamiltonian systems form the cornerstone of classical mechanics, unifying kinetic and potential energy within the Hamiltonian function:

$$H(q, p) = T(p) + V(q),$$

where  $q$  and  $p$  represent position and momentum, respectively. The evolution of these quantities is governed by:

$$\dot{q} = \frac{\partial H}{\partial p}, \quad \dot{p} = -\frac{\partial H}{\partial q}.$$

This framework is geometrically interpreted using phase space, a  $2n$ -dimensional manifold combining  $q$  and  $p$ . Recent advancements propose integrating dual vector algebra with Hamiltonian mechanics, providing a compact representation of position, momentum, and their derivatives.

## 2 Symplectic Geometry and Hamiltonian Systems

Symplectic geometry underpins Hamiltonian mechanics, introducing a symplectic form:

$$\omega = dq \wedge dp.$$

This form preserves phase-space volume, as stated in Liouville's theorem, and enables the expression of Hamiltonian equations via:

$$\iota_{X_H} \omega = dH,$$

where  $X_H$  is the Hamiltonian vector field. Symplectic transformations, defined by:

$$T^T J T = J, \quad J = \begin{bmatrix} 0 & I \\ -I & 0 \end{bmatrix},$$

preserve the symplectic structure and phase-space dynamics.

### 3 Dual Numbers and Their Algebra

Dual numbers extend real numbers through:

$$A = a + \varepsilon a^*, \quad \varepsilon^2 = 0.$$

Key operations include:

- Addition:  $(a + \varepsilon a^*) + (b + \varepsilon b^*) = (a + b) + \varepsilon(a^* + b^*)$ .
- Multiplication:  $(a + \varepsilon a^*)(b + \varepsilon b^*) = ab + \varepsilon(a^*b + ab^*)$ .

These properties simplify the representation of translations and derivatives. For example, a dual parametric function  $X = x + \varepsilon x^*$  expands as:

$$f(X) = f(x) + \varepsilon x^* \frac{df(x)}{dx}.$$

### 4 Dual Vectors and Their Applications

A dual vector in  $\mathbb{R}^3$  is defined as:

$$\mathbf{R} = \mathbf{q} + \varepsilon \mathbf{p},$$

where  $\mathbf{q}$  represents position and  $\mathbf{p}$  represents momentum. This compact form facilitates the analysis of motion and kinematics. Applications include:

- Screw motion: Combines rotation and translation in robotics.
- Ruled surfaces: Represented as dual parametric vector functions.

#### 4.1 Ruled Surfaces

A dual parametric vector function is expressed as:

$$\mathbf{R}(T) = \mathbf{q}(t) + \varepsilon \mathbf{q}'(t)t^*,$$

where  $\mathbf{q}(t)$  is a unit curve. For example:

$$\mathbf{q}(t) = (\cos t, \sin t, 0), \quad \mathbf{q}'(t) = (-\sin t, \cos t, 0).$$

The corresponding ruled surface is:

$$\mathbf{R}(t, \lambda) = \lambda \mathbf{q}(t) + \mathbf{c}(t),$$

where  $\mathbf{c}(t)$  is a base curve. Applications include modeling conical and pendulum-like surfaces.



## 5 Hamiltonian Systems and Dual Vectors

The dual vector framework aligns naturally with Hamiltonian phase spaces. A 3D dual parametric vector function corresponds to a 6D Hamiltonian system:

- Real part  $\mathbf{q}(t)$ : Position.
- Dual part  $\mathbf{q}'(t)$ : Momentum.

This correspondence provides a unified framework for describing dynamics, enabling new insights into energy conservation and motion trajectories.

## 6 Applications and Future Directions

Dual vectors have broad applications:

- **Robotics:** Precise motion control and screw motion modeling.
- **Mechanics:** Unified representation of forces and torques.
- **Computer Graphics:** Dynamic surface modeling.

Future research will explore:

- Extending dual vector theory to higher dimensions.
- Investigating computational trade-offs in large-scale systems.
- Developing visualization tools for extended phase spaces.

## 7 Conclusion

Dual vectors offer a compact and powerful representation for physical systems, bridging geometric and algebraic interpretations. Their integration with Hamiltonian mechanics and symplectic geometry promises to enhance both theoretical and practical approaches to dynamics.

### Acknowledgments

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## 2D BOSONIC OSCILLATOR IN A MAGNETIC FIELD IN NONCOMMUTATIVE ANTI DE-SITTER SPACES

Lakhdar **SEK**

NTDL Laboratory, Faculty of Technology, University of El Oued, 39000 El Oued, Algeria.  
0009-0004-0582-5949  
sek-lakhdar@univ-eloued.dz

Mokhtar **FALEK**

Department of Matter Sciences, University of Khenchela, 40000 Khenchela, Algeria.  
0000-0002-0466-9559  
falek.mokhtar@univ-khenchela.dz

Mustafa **MOUMNI**

Laboratory of Radiation Physics and their Interactions with Matter (LPRIM), Department of Physics, University  
of Batna 1, 05000 Batna, Algeria.  
0000-0002-8096-6280  
m.moumni@univ-batna.dz

### Abstract

In this paper, we study (2+1) D Klein Gordon oscillator in a uniform magnetic field using the non-commutative and Anti de-Sitter spaces. The energy eigenvalues are given in their exact forms, and the corresponding radial wave functions are expressed with associated Jacobi polynomials. We determine the critical magnetic field value, which cancels out the normal Zeeman effect. We have also looked into how our system's thermodynamic properties are affected by the spatial deformation parameters, we remark that the graphs of these quantities show that all thermodynamic quantities, with the exception of the Helmholtz free energy, have an inverse proportion with the deformation parameters  $(\theta; \lambda)$ . For every figure, it appears that the noncommutative effect is generally much smaller than the AdS's.

## A NEW APPROACH TO EXPLORE QUASINORMAL MODES OF BLACK HOLES

Narges **HEIDARI**

Faculty of Physics, Shahrood University of Technology  
Shahrood, Iran  
0000-0002-4623-8909  
heidari.n@gmail.com

Hassan **HASSANABADI**

Faculty of Physics, Shahrood University of Technology  
Shahrood, Iran  
0000-0001-7487-6898  
hha1349@gmail.com

### Abstract

This study proposes a novel analytical framework for the determination of Quasi Normal Modes (QNMs) associated with black holes. The method utilizes the Rosen-Morse function to derive the approximate quasi-normal frequencies for the Schwarzschild black hole. The approach presented herein, when compared with prior methodologies, exhibits a higher degree of accuracy in its approximations. The refined modes identified through this investigation are posited to represent a generalized variant of QNMs obtained using the Pösch-Teller function. The numerical outcomes of this innovative approach are computed and contrasted with those derived from the Mashhoon method and other numerical techniques.

## THEORETICAL INVESTIGATION OF THE BULK PHOTOVOLTAIC EFFECT IN MIXED HALIDE PEROVSKITES FOR SOLAR CELLS

Naouel **CHELIL**

Laboratoire de Physique Quantique de la Modélisation Mathématique (LPQ3M), Mascara, Algeria.  
0000-0001-5911-8087  
n.chelil@univ-mascara.dz

Mohammed **SAHNOUN**

Laboratoire de Physique Quantique de la Modélisation Mathématique (LPQ3M), Mascara, Algeria.  
0000-0001-5200-0254  
msahnoun@univ-mascara.dz

### Abstract

Photoexcitation in solids can induce electron-hole transitions between electronic bands, and in materials lacking inversion symmetry, these transitions can generate a spontaneous photocurrent due to the Berry connection, known as shift current. This effect is a promising alternative to traditional p-n junction solar cells. Lead-free halide perovskites, particularly  $CsGeI_2Cl$ , are highlighted for their potential in this area. In this work, based on density functional theory calculations,  $CsGeI_2Cl$  with its hexagonal structure and absence of inversion symmetry, exhibits a moderate spontaneous polarization of  $17.71 \mu C.cm^{-2}$  and a direct band gap of about  $1.75 eV$ , which are conducive to charge separation. Our first-principles calculations reveal that  $CsGeI_2Cl$  can produce a significant photocurrent in the visible range, tripling the response of  $BiFeO_2$ , making it a strong candidate for renewable energy applications due to its excellent ferroelectric and nonlinear optical properties.

## NANOFIBRES FOR WOUND DRESSING APPLICATION

Faiza **BETTAHAR**

Laboratoire de Chimie des Polymères, Université Oran1 Ahmed Ben Bella, El-Mnaouer, BP 1524, Oran 31000, Algeria.

0000-0001-7644-6082

faiza-bettahar@hotmail.com

### Abstract

Electrospinning was used to examine the fiber development of alternating polymer based vinyl acetate produced by free radical polymerization (AIBN, 1h) from different solutions. Toluene, styrene, benzene, dimethylformamide (DMF), and tetrahydrofuran (THF) all dissolved polymers [1].

Polymers fibers may be electrospun using solutions containing 15% to 20% DMF and 20% THF. The findings demonstrated that the fiber generation from electrospinning was significantly impacted by solvent characteristics and polymer-solvent miscibility. In general, polymers fiber diameters decreased and their homogeneity was enhanced by the addition of solvents such THF was used for wound dressing application causing the VAMA fiber diameters to drop dramatically to 200 or 300 nm [2].

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## MODELING THE DIFFUSION OF OPINION ON CLIMATE CHANGE: A SIMPLIFIED PRACTICE OF SOCIAL PHYSICS

Ali **AKYÜZ**

Burdur Mehmet Akif Ersoy University, Bucak Emin Gulmez Vocational School of Technical Sciences, Burdur,  
Turkey

0000-0001-9265-7293

aliozhanakyuz@gmail.com

Elif P. **TUNCER**

Istanbul University, Open and Distance Education Faculty, Istanbul, Turkey

0009-0008-3801-7453

eptuncer@gmail.com

Yesim **OKTEM**

Istanbul University, Faculty Of Science, Department Of Physics, Istanbul, Turkey

0000-0002-1638-4331

yesim.oktem@istanbul.edu.tr

### Abstract

This study focuses on the processes of climate change belief and non-belief in social networks under the influence of media, leadership and community structures. Social structure was represented by the Barabási-Albert model. This model successfully modeled interactions between individuals with leadership influence and other individuals, reflecting the distribution of connections in social systems. The model examined the influence of two main elements: leader influence and media influence. Leader influence represents the power of individuals with strong connections within the network to spread their views to those around them, while media influence deals with how external messages are disseminated to society. In the simulation, individuals' opinions were constantly updated through a combination of media messages, leader influence and social interactions. In addition, the Louvain method was used to identify communities in the network and analyze how climate change belief and denial spread within different groups. The results reveal the decisive role of media and leader influences in shaping opinions.

# 1 Introduction

Sociophysics is an interdisciplinary science that uses physical and mathematical methods to understand social phenomena and human behavior. This field aims to analyze social dynamics by modeling the interactions of individuals and groups. In particular, big data analytics, network theory and statistical physics methods are among the basic tools of sociophysics. Sociophysics has an important role in understanding decision-making processes, diffusion dynamics and collective behavior in society. The discipline offers a more inclusive perspective by seeking to explain the order and chaos underlying social systems.

The dynamics of opinion formation in interconnected societies have become increasingly important in an era defined by rapid information exchange. Social networks, both physical and virtual, serve as conduits for the dissemination of ideas, beliefs and behaviors. Within these networks, individuals' opinions are shaped not only by interpersonal interactions, but also by the influence of authoritative leaders and ubiquitous media [1–4].

Simulating opinion dynamics allows researchers to explore complex phenomena such as polarization, consensus formation and knowledge cascades. This study uses a scale-free network model to simulate a realistic social connection structure in which influential individuals play disproportionate roles. The addition of heterogeneous influence coefficients - capturing variations in sensitivity to media and peers - improves the accuracy of the model.

This study investigates the interplay between media messages, leader influence, and community structures that shape network-wide opinions. By combining computational modeling and community detection methods, the study highlights patterns of opinion diffusion and the emergence of collective behaviors [5–7].

# 2 Methodology

The simulation used in this study is built on a scale-free network structure that mimics real-world social networks, defined by a few highly connected nodes (hubs) generated with the help of the Barabási-Albert algorithm. In this network, nodes represent individuals and edges represent social connections between individuals. The simulation is governed by three key parameters: Media Influence ( $\mu$ ) reflects the extent to which individuals are influenced by external media messages. Leader Influence ( $\lambda$ ) refers to the influence of the views held by identified influential individuals (leaders) on other individuals. Finally, Random Turns is a stochastic element that allows for random changes of opinion with a small probability, increasing the variability in the system and modeling the sudden changes of opinion of real-life individuals [8,9].

Each individual's opinion is updated based on a weighted combination of media messages, the opinions of their neighbors and the individual's own internal biases. This mechanism allows individuals to be influenced by the opinions around them through social interactions, while emphasizing the role of media messages in individual opinions. Leaders, in particular, are modeled to be more likely to adhere to media messages. By combining deterministic and random factors, the opinion updating rule allows individuals to update their opinions dynamically and heterogeneously over time [10,11].

The identification of communities within the network is performed using the Louvain method, which optimizes modularity to group nodes into coherent clusters. This method allows the identification of community structures based on the intensity of interactions between individuals in the network. This allows for a detailed analysis of how ideas evolve within and across community structures. This approach provides a powerful tool for studying the tendency of individuals to reach consensus within homogeneous groups and possible divergences between different communities [12].

### 3 Results and Discussion

Throughout the 50 simulation steps, the percentage of positive and negative opinions fluctuated in response to media influence and leader-driven interactions. In networks where leaders were strongly aligned with the media message, positive opinions dominated. In contrast, in the absence of strong leader alignment, opinions were often polarized and different communities showed opposing trends.

Figure 1 shows the spatial trends in favorable and unfavorable opinions across the network. (a) at the beginning of the simulation and (b) at the end of 50 simulation steps. The observed oscillations emphasize the dynamic interaction between external influence (media) and internal network feedback. In Figure 1, nodes represent individuals in the network. Red nodes represent individuals who believe in climate change, while blue nodes represent individuals with opposing views. The lines show the interactions between individuals. In Figure 1, we first present the situation at the beginning of a process (time step 1). The initial distribution of the two different views (red and blue) in the network seems to be random and they are equally distributed.

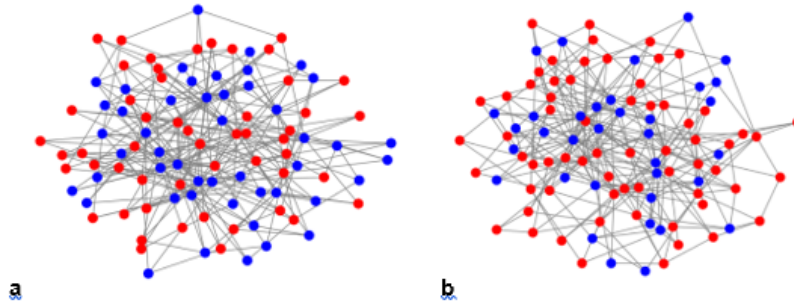


Figure 1: (a) Opinion spread for time step 1 (b) for time step 50.

The network seems to be quite dense. Each individual is connected to many people. Red and blue opinions are not homogeneously distributed and there are clusters in some areas: The number of red nodes has increased significantly over time due to the density of connections. This shows that the red view has become more widespread over time. Blue nodes are still present but have decreased, suggesting that the number of individuals with a blue view has changed over time, influenced by time. The network structure maintains the density of connections. As a result of interactions, more individuals (nodes) seem to have switched to red vision. Since the nodes in the center of the network are more densely connected, opinion diffusion may have occurred faster in the central regions. These nodes could be influential actors that influence other individuals. The random distribution of red and blue opinions at timestep 1 is unbalanced at timestep 50, with red becoming dominant. The spread of opinions is most likely driven by interactions between neighbors and local majority influence.



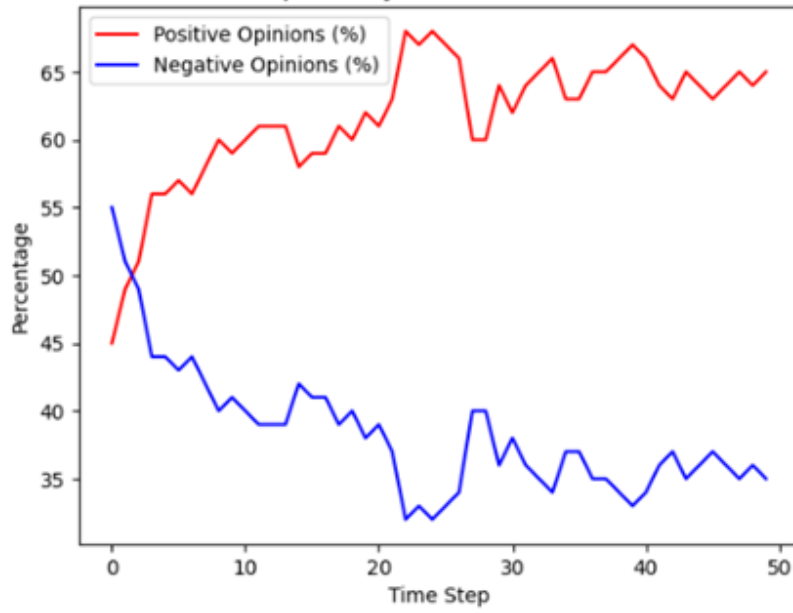


Figure 2: Opinion Dynamics over time.

Figure 2 illustrates the dynamics of views on climate change over time. Positive views, i.e. those who believe in the existence of climate change, initially constituted 45% of the society, while negative views, i.e. those who do not believe in climate change, were dominant at 55%. Over time, however, there has been a noticeable increase in the proportion of individuals who accept the effects of climate change, reaching 65%. In contrast, there has been a marked decline in the proportion of deniers, stabilizing at around 35% over time. This change can be explained by various factors. The increase in scientific research and concrete evidence on climate change, more awareness-raising activities by the media, and especially the fact that environmental disasters are felt in people's daily lives may have changed the attitudes of individuals. The fact that positive views are becoming more dominant shows that the society is making progress in the process of awareness raising. As a result, it shows how public awareness can change over time and that belief in climate change is becoming a widespread view. However, it should not be forgotten that 35% still do not accept this reality. This points to the need to further increase information efforts on climate change through education, media and policies. Adding random changes of opinion with a probability of 5% to the system prevented the formation of early consensus, preserving diversity of opinion and adding a certain level of noise to the system. This reflects real-world scenarios where individuals change their beliefs about climate change due to unforeseen events, new information or personal experiences. The randomness suggests that changes of opinion are not entirely systematic and that external factors can influence whether people believe or disbelieve in the reality of climate change.

The findings of this study reveal that leaders, community structures and communication dynamics play a critical role in building consensus on opinion-based issues such as climate change. The applications of the study can benefit different sectors. For example, in terms of policy design, understanding how effective climate change awareness campaigns can promote consensus among the public based on scientific facts. In the context of marketing, analyzing the dynamics of public opinion on climate change can contribute to the development of targeted strategies for sustainable products and green campaigns. Social media management reveals the opportunity for platforms to

reduce polarization by identifying groups resistant to misinformation, increasing access to accurate information and promoting cross-community communication. The study also emphasizes that the transformation of societies into echo chambers can increase polarization on climate change. In this context, it is argued that interventions that promote dialogue between groups with different views have the potential to build broad consensus. Such interactions can help society develop a shared attitude towards the reality of climate change.

## 4 Conclusion

This simulation study reveals the complex interplay between media, leadership, and community structures in shaping opinion dynamics within social networks. Scale-free network modeling, combined with community detection, provides a powerful framework for exploring these interactions. The results demonstrate how small variations in influence coefficients or random opinion flips can lead to vastly different outcomes, from consensus to polarization. Future research should explore multi-layered networks and incorporate real-world data to refine these models further.

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